CBSE 12th Mathematics
Chapter 1 (Relations and functions)
Important Questions Unsolved

SECTION - A

Question number 1 to 13 carry 1 mark each.

Q.1: If a*b denotes the larger of 'a' and 'b and if a o b = (a*b) + 3, then write the value of (5) o (10), where * and o are binary operations.

Q.2: Let * be a binary operation defined by a * b = 2a + b -3. Find 3 * 4.

Q.3: Let * be a binary operation on N given by $a * b = HCF\ (a, b)\ a, b\ \in N$. Write the value of $22 * 4$.

Q.4: If: $R \to R$ be defined by $f(x) = (3 - x^3)^{1/3}$, then find f of (x).

Q.5: Let $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. State whether f is one-one or not.

Q.6: The binary operation *: R x R → R, is defined as a*b = 2a + b. find (2*3)*4.

Q.7: If $(R) = \{(x, y): x + 2y = 8\}$ is a relation on N, write the range of R.

Q. 8: If $f(x) = x + 7$ *and* $g(x) = x - 7, x \in R$,

find (fog) (7).

Q.9: If binary operation * on the set of integers Z, is defined by $a * b = a + 3b^2$, then find the value 2 * 4.

Q. 10: What is the range of the function:

$$f(x) = \frac{|x - 1|}{(x - 1)}?$$

Q.11: State the reason for the relation R in the set $\{1, 2, 3\}$ given by R = $\{(1, 2), (2, 1)\}$ not to be transitive.

Q.12: Let * be a 'binary' operation on N given by $a * b = LCM\ (a, b) \forall a, b \in$ N.

Q.13: Let * be a binary operation, on the set of all non- zero real number, given by

$$a * b = \frac{ab}{5}\ for\ a, b \in R - \{0\}.$$

Find the value of x, given that $2 * (x * 5) = 10$.

SECTION - B

Question number 14 to 31 carry 4 mark each.

Q.14: Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$ such that g of $= fog = I_R$.

Q.15: A binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as

$$a * b = \begin{cases} a + b, & \text{if} \quad a + b < 6 \\ a + b - 6 & \text{if} \quad a + b \geq 6 \end{cases}$$

Show that zero is the identify for this operation and each element $'a'$ of the set is invertible with $6 - a$, being the inverse of $'a'$.

Q. 16: (i) Is The binary operation *, defined on set N, given by

$a * b = \frac{a+b}{2}$ $for\ all\ a, b \in Q, commutative?$ (ii) is the above binary operation * associative?

Q. 17: Prove that the relation R in the set

$A = \{1, 2, 3, 4, 5\}$ *given by* $R = \{(a, b) : |a - b|$ *is even*$\}$, *is an equivalence relation.*

Q.18: Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b) : a, b \in Z$ and $(a - b)$ is divisible by $5.\}$ Prove that R is an equivalence relation.

Q.19: Consider the binary operation * on the set {1, 2, 3, 4, 5} defined by

$a * b = \min\{a, b\}$. Write the operation table of the operation *.

Q.20: Let A = IR – {3} and B = IR – {1}. Consider the function $f A \to B$ defined by

$f(x) = \left(\frac{x-2}{x-3}\right)$. show that f is one – one and onto and hence find f^{-1}

Q. 21: *Let* $A = \{1, 2, 3, \ldots\ldots,\}$ and R be the relation in A × A defined by $(a, b)\ R\ (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ *in* A × A. Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)].

Q. 22: Show that the function f *in* $A = R - \left\{\frac{2}{3}\right\}$

Defined as $f(x) = \frac{4x+3}{6x-4}$ *is one – one and onto. hence find* f^{-1}.

Q.23: Show that the relation R defined by (a, b) R (c, d) ⇒a + d = b + c on the set N x N is an equivalence relation.

Q.24: Let $f: N \to N$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in \mathrm{N}.$$

Find whether the function f is bijective.

Q.25: Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b): a, b \in Z, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

Q.26: Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$ such that $gof = fog = I_R$.

Q.27: A binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as

$$a * b = \begin{cases} a + b, & \text{if } \ a + b < 6 \\ a + b - 6 & \text{if } \ a + b \ge 6 \end{cases}$$

Show that zero is the identity for this operation and each element $'a'$ of the set is invertible with $6 - a$, being the inverse of $'a'$.

Q.28: **Show that $f: N \rightarrow N$, given by,**

$$f(x) = \begin{cases} x+1, & if \quad x \text{ is odd} \\ x-1, & if \quad x \text{ is even} \end{cases} \quad \textit{is both one} - \textit{one and onto.}$$

Q.29: **Consider the binary operations* : R x R →R defined as a * b = $|a - b|$ and a o b = a for all a, b ϵ R. show that '*' is commutative but not associative 'o' is associative but not commutative.**

Q.30: **Consider $f: R_+ \rightarrow [4, \infty)$given by $f(x) = x^2 + 4$.Show that f is invertible with the inverse f^{-1} of given by f^{-1} (y) =$\sqrt{y-4}$ where $R_+$is the set of all non – negative real numbers.**

Q.31: **If the function $f: R \rightarrow R$ be given by $f(x) = x^2 + 2$ and $g: R \rightarrow R$ be given by**

$$g(x) = \frac{x}{x-1}, x \neq 1,$$

find fog *and* gof and hence find $fog(2)$ *and* $gof(-3)$.

SECTION - C

Question number 32 to 42 carry 6 mark each.

Q.32: Consider $f: R \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$.

Show that f is invertible with $f^{-1}(y) = \left(\sqrt{\frac{y+6-1}{3}}\right)$

Hence Find

(i) $f^{-1}(y)$

(ii) y if $f^{-1}(y) = \frac{4}{3}$,

Where R_+ is the set of all non-negative real numbers.

Q.33: Discuss the commutativity and associativity of binary operation '*' defined on $A = \ Q - \{1\}$ by the rule $a * b = a - b + ab$ for all $a, b \in$ A. Also find the identity element of * in A and hence find the invertible elements of A.

Q.34: Let N denote the set of all natural numbers and R be the relation on N x N defined by (a, b) R (c, d) if ad (b + c) = bc (a + d). Show that R is an equivalence relation.

Q.35: *Let* $f: N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$.

Show that $f: N \to S$,

Where S is the range of f,is invertible. Find the inverse of f and hence find $f^1(43)$ and $f^1(163)$

Q.36: Let $A = \{x \in Z: 0 \le x \le 12\}$. Show that $R = \{(a, b): a, b \subset A, |a\ b|\ is\ divisible\ by\ 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class[2].

Q.37: Show that the function $f: \mathbb{R} \to \mathbb{R}$ defined by

$f(x) = \dfrac{x}{x^2 + 1}, \forall x \in \mathbb{R}$ neither one – one nor onto.

Also, if $g: \mathbb{R} \to \mathbb{R}$ is defined as $g(x) = 2x - 1$, find $fog(x)$.

Q.38: Consider $f: R — \frac{4}{3} \to R - \left\{\frac{4}{3}\right\}$ *given by*

$f(x) = \dfrac{4x + 3}{3x + 4}$ *Show that f is bijective.*

Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

Q.39: Let A= Q × Q and let * be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)\ for\ (a, b), (c, d) \in A$. Determine, whether * is commutative and associative. Then, with respect to * on A

(i) Find the identity element in A.
(ii) Find the invertible elements of A.

Q.40: Consider$f: \mathbb{R}_+^{\to}[-9\infty]$give by $f(x)5x^2 + 6x - 9$. Prove that f is invertible with:

$$f^{-1}(y) = \left(\frac{\sqrt{54 + 5y} - 3}{5}\right)$$

Q.41: A binary operation * is defined on the set $X = R - \{-1\}$ *by* $x * y = x + y + xy, \forall x, y \in X$.Check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X.

Q.42: Show that the binary operation * on A = R – {-1} defined as a * b = a + b + ab for all a, b, ε A is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible.